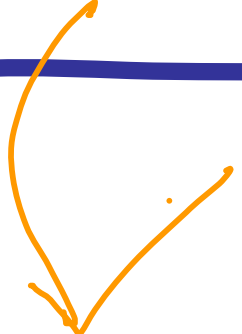


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MUTUAL INFORMATION


$$\begin{aligned} I(X;Y) &= H(X) - H(X|Y) \\ &= H(Y) - H(Y|X) \\ &= I(Y;X). \end{aligned}$$

Example 1:

		$\frac{1}{2}$ r_0	$\frac{1}{4}$ r_1	$\frac{1}{8}$ r_2	$\frac{1}{8}$ r_3
$\frac{1}{4}$	s_0	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{4}$	s_1	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{4}$	s_2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$\frac{1}{4}$	s_3	$\frac{1}{4}$	0	0	0

$$H(X) = 2 \quad H(Y) = 1.75 = \frac{7}{4}$$

$$H(X, Y) = 3.375 = \frac{27}{8}$$

$$H(Y|X) = 1.375 = \frac{11}{8}$$

$$\begin{aligned} I(X;Y) &= H(Y) - H(Y|X) \\ &= \frac{7}{4} - \frac{11}{8} \end{aligned}$$

$$\boxed{I(X;Y) = \frac{3}{8}}$$

$$\begin{aligned} H(Y) + H(X|Y) &= H(X,Y) \\ \Rightarrow H(X|Y) &= \frac{27}{8} - \frac{7}{4} \end{aligned}$$

$$H(X|Y) = \frac{13}{8}$$

conditional
Entropy of
 X given Y .

$$H(X) - H(X|Y)$$

$$I(X;Y) = 2 - \frac{13}{8} = \frac{3}{8}$$

$$I(X;Y) = H(X) + H(Y) - H(X;Y)$$

$$= 2 + \frac{7}{4} - \frac{27}{8}$$

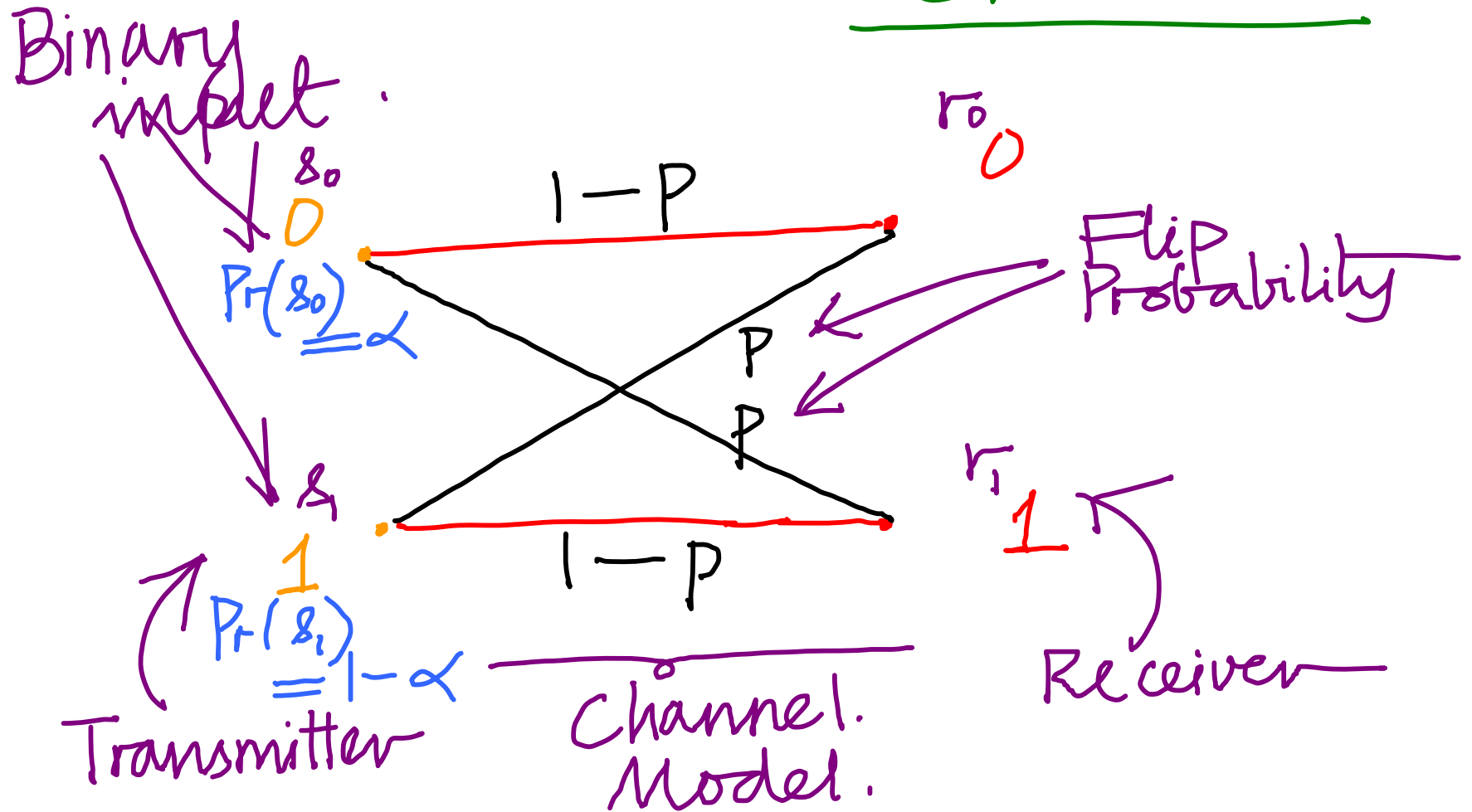
$$= \frac{15}{4} - \frac{27}{8}$$

$$I(X;Y) = \frac{3}{8}$$

Mutual
Information between
 X, Y .

Example:

Binary Symmetric Channel:



$$P(r_0|s_0) = P(r_1|s_1) = 1 - P$$

$$P(r_1|s_0) = P(r_0|s_1) = P$$

Flip
Probabilities.

$$Pr(X = s_0) = \alpha$$

$$\Rightarrow Pr(X = s_1) = 1 - \alpha.$$

conditional
Entropy.

$$H(Y|X) = Pr(X = s_0) H(Y|X = s_0) + Pr(X = s_1) H(Y|X = s_1)$$

Given $X = s_0$;

$$\left. \begin{aligned} \Pr(Y = r_0 | s_0) &= 1 - P \\ \Pr(Y = r_1 | s_0) &= P \end{aligned} \right\}$$

$$H(Y|X = s_0)$$

$$= H(P, 1-P)$$

$$= H(P).$$

$$= P \log_2 \frac{1}{P}$$

$$+ (1-P) \log_2 \left(\frac{1}{1-P} \right)$$

Given $X = s_1$.

$Y = r_1$ with prob $1-P$

$Y = r_2$ with prob P .

$$H(Y|X=z_1) = H(p).$$

$$\begin{aligned}
 H(Y|X) &= \underbrace{Pr(X=z_0)}_{\alpha} \cdot \overbrace{H(Y|X=z_0)}^{H(p)} \\
 &\quad + \underbrace{Pr(X=z_1)}_{1-\alpha} \cdot \overbrace{H(Y|X=z_1)}^{H(p)} \\
 &= \alpha H(p) + (1-\alpha) H(p) \\
 &= H(p).
 \end{aligned}$$

$$H(Y|X) = H(p)$$

↑
conditional Entropy of
Transmitted symbol Y given X

$$H(\cancel{X}) = H(\alpha).$$

$$\begin{aligned} \Pr(r_0) &= \Pr(r_0 \cap s_0) \\ &\quad + \Pr(r_0 \cap s_1) \\ &= \frac{1-p}{\Pr(r_0|s_0)} \cdot \Pr(s_0) \\ &\quad + \frac{\Pr(r_0|s_1)}{p} \cdot \frac{\Pr(s_1)}{1-\alpha} \\ &= (1-p)\alpha + p(1-\alpha) \end{aligned}$$

$$\boxed{\Pr(Y=r_0) = \alpha + p - 2\alpha p.}$$

$$\begin{aligned} \Pr(Y=r_1) &= 1 - \Pr(Y=r_0) \\ &= 2\alpha p - \alpha - p + 1 \end{aligned}$$

$$H(Y) = H(\alpha + p - 2\alpha p)$$

$$I(X; Y) = H(Y) - H(Y|X)$$

$$I(X; Y) = H(\alpha + p - 2\alpha p) - H(p)$$

mutual Information for BSC
 Flip Prob = p
 Source Prior Prob = $\alpha, 1 - \alpha$.